Biomechanical modeling of articular cartilage replacement materials


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Motivation and aim

- Replacement of human articular cartilage by *condensed cellular collagen gel*.
- Development of a viscoelastic-diffusion and remodeling model for cellular material.
- Theoretical and experimental study of remodeling effects.
Contents

1. Bioreactor
2. Theoretical model
3. Finite element model
4. Compression tests
5. Theoretical / Experimental comparison
6. Conclusions
1. Bioreactor

- Cyclic mechanical stimulation in a bioreactor.
Histological evaluations

Unstimulated control sample

Stimulated bioreactor sample

Indistinguishable and little growth of collagen type-II

Discernible and more growth of collagen type-II
Phenomenon in bioreactor

Initial configuration

Current configuration

stimulation in bioreactor

weight function

cell

remodeling effect due to synthesized fiber

condensed collagen gel

stimulation

no remodeling effect

$\sigma^r$, $\sigma^e$, $\sigma^{ve}$, $\sigma^d$
2. Theoretical model

Cauchy stress rate tensor:

\[ \dot{\sigma} = \dot{\sigma}^e + \dot{\sigma}^{ve} + \dot{\sigma}^d + \dot{\sigma}^r \]

- Elastic part:
  \[ \sigma^e = C : \varepsilon \]
  (\(C\) : elasticity tensor)

- Viscoelastic part:
  \[ \dot{\sigma}^{ve} = \tilde{C} : \dot{\varepsilon} \]
  (\(\tilde{C}\) : visco-elasticity tensor)

- Diffusive part:
  \[ \dot{\sigma}^d = -D\sigma \]
  (\(D\) : diffusion parameter)

- Remodeling part:
  \[ \dot{\sigma}^r = C^r : \dot{\varepsilon} \]
  (\(C^r\) : remodeled elasticity tensor)
Diffusion

Diffusive part:

\[ \dot{\sigma}^d = -D\sigma \]

\[ \dot{\sigma}^d_1 = \dot{\sigma}^d_3 = -D\sigma_1 \quad \text{and} \quad \dot{\sigma}^d_4 = \dot{\sigma}^d_5 = \dot{\sigma}^d_6 = 0 \]

with

\[ D(\varepsilon_v) = D_0 + D_1 \varepsilon_v \quad (\varepsilon_v \text{- volumetric strain}) \]
Remodeling part:

\[ \dot{\sigma}^r = \mathbf{C}^r : \dot{\varepsilon} \]

Evolution equation:

\[ \dot{\mathbf{C}}^r = \Phi k \sqrt{\psi} (\mathbf{C}^r_{\text{crit}} - \mathbf{C}^r) \]

\[ \rightarrow \quad \dot{C}^r_{11} = \dot{C}^r_{33} = \Phi(x_2) k \sqrt{\psi} (C^r_{11,\text{crit}} - C^r_{11}) \]

with

- \( \Phi \) - weight function of cell distribution
- \( k \) - remodeling velocity parameter
- \( C^r_{11,\text{crit}} \) - limit value of stiffness component
- \( C^r_{11} \) - stiffness change due to remodeling effect
Free energy

\[ \psi = \frac{1}{2} \lambda \ln^2 J + \frac{1}{2} \mu (\mathbf{F}^T \mathbf{F} : \mathbf{I} - 3) - \mu \ln J + \int \sigma^r : \mathbf{d} \varepsilon \]

- isotropic Neo-Hookean-type free-energy function
- deformation energy due to newly synthesized collagen fiber

\[ \rightarrow \psi = \frac{1}{2} \lambda \ln^2 J + \frac{1}{2} \mu (\mathbf{F}^T \mathbf{F} : \mathbf{I} - 3) \]

\[ - \mu \ln J + \int \sigma^r_{11} d \varepsilon_{11} + \int \sigma^r_{33} d \varepsilon_{33} \]

with \( J = \text{det} (\mathbf{F}) \) and \( \lambda, \mu \) - Lamé constants
3. Finite element model

- Stress increment in user defined subroutine Umat in Abaqus:

\[
\Delta \sigma_{ij} = C_{ijkl} \Delta \varepsilon_{kl} + \tilde{C}_{ijkl} \Delta \varepsilon_{kl} + \Delta \sigma^d_{ij} + C^{r}_{ijkl} \Delta \varepsilon_{kl}
\]

short time scale / sec  long time scale / week

- Gradient material (Linear distribution of stiffness):

\[
c_{11}(x_2) = c_{11,i} + \frac{c_{11,a} - c_{11,i}}{h} \cdot x_2
\]
4. Compression tests

- MTS Material testing machine
5. Theoretical / Experimental comparison

Before stimulation in bioreactor

\[ E_{11} = E_{22} = 0.8 \text{ MPa} \]
\[ D_0 = 2.58 \text{ 1/s} \]
\[ D_1 = 2.7 \text{ 1/s} \]

After stimulation in bioreactor

\[ E_{11} \rightarrow 1.2, E_{22} = 0.8 \text{ MPa} \]
\[ D_0 = 26.1 \text{ 1/s} \]
\[ D_1 = 22 \text{ 1/s} \]
Remodeling of stiffness

initial state

after 4 days

after 9 days

after 14 days

after 18 days

after 23 days

after 28 days

\[ C_{11} + C_{11}^r \]
or
\[ C_{33} + C_{33}^r \]

SDV3 (Avg. 75%)

+2.001e+05
+1.935e+05
+1.869e+05
+1.803e+05
+1.736e+05
+1.670e+05
+1.604e+05
+1.538e+05
+1.472e+05
+1.406e+05
+1.339e+05
+1.273e+05
+1.207e+05
6. Conclusions

• A remodeling phenomenon in bioreactor was investigated and a constitutive model was proposed.

• The replacement material exhibits high strength that is experimentally and theoretically studied.

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